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## STUDYING THE DISTURBANCES OF ROBOTIC ARM MOVEMENT IN SPACE USING THE COMPOUND-PENDULUM METHOD

## Introduction

In this research, we suppose the representation of the arm movement with a related payload as a compound-pendulum with the adoption of a mathematical procedure [1-4]. This gives a complete and clear picture of the disturbances that are affecting the inertia moment and center of gravity (CoG) in the coordinate frame of the aircraft during its movement in the air. This method, which was not mentioned in literature before gives a complete handling of noise determination and formulates random disturbances' functions [5] that can be emulated and inserted into the dynamics model of the aircraft. Inertial moments amounts were studied according to the overall aircraft model [6, 8-10], in addition to applying the theory of parallel axis of HuygensSteiner [7], which is concerned with studying the new inertia moment of the studied part relative to new axis of study parallel to the axis of the part to be studied.

Since the used aircraft model is a multirot or UAV model, which is considered a solid body with a symmetric form, the parameters of the change in the distur-
bances functions of the motion equations can be defined according to the general form based on the pendulum model shown in equation (1).The shape of the aircraft was supposed approximately as a rectangle as shown in figure (1). The pendulum motion takes place according to the angles of motion $\left(\theta_{1}, \theta_{2}, \theta_{3}\right)$ as illustrated in the figure and these angles ranges are: $\left(0<\theta_{1}<360\right.$, $-5<\theta_{2}<-175,-155<\theta_{3}<155$ ), assuming the weight of the payload is fixed. This will lead to the following parameters change: the center of gravity of the aircraft dynamics model and the overall inertia moment of the aircraft in addition to changes in the thrust resulting from the aerial motors because of the distance change between the center of gravity of the aircraft dynamics model and each engine. The general form of disturbances can be expressed as follow [5]:

$$
\begin{equation*}
F_{\text {dist }}=f_{F}\left(\theta_{1}, \theta_{2}, \theta_{3}\right), M_{\text {dist }}=f_{M}\left(\theta_{1}, \theta_{2}, \theta_{3}\right), \tag{1}
\end{equation*}
$$

where $f_{F}$ and $f_{M}$ are nonlinear stochastic functions.


Fig. 1. General model of the pendulum movement disturbances

[^0]The simplification is based on the assumption that the pendulum motion occurs when the aircraft is fixed in the air in hovering position, therefore the aerodynamic effects resulting from the airflow through the pendulum become neglected.

In the following, the change in the overall center of gravity and the inertia moment resulting from the pendulum motion will be studied mathematically and will be compared with the measurements resulting from the simulation depending on SolidWorks and MATLAB software.Assuming the initial values of the pendulum angles are:

$$
\left(\theta_{1}=0^{\circ}, \theta_{2}=-5^{\circ}, \text { and } \theta_{3}=-155^{\circ}\right),
$$

and according to the physical characteristics defined in figure 1 , the pendulum parts are defined by the characteristics in table (1), where the overall center of gravity of the aircraft is defined by the $X B, Y B, Z B$ coordinates. While each part of the pendulum has a local coordinate system, coinciding with the local center of gravity related to that part as shown in figure (2) in blue in comparison with the aircraft's overall center of gravity marked in black and white.

Table 1. General Specifications of the overall aircraft model

| $M_{b}$ | $M_{j}$ | $M_{l 1}$ | $M_{l 2}$ | $M_{l}$ |
| :---: | :---: | :---: | :---: | :---: |
| $25,0 \mathrm{Kg}$ | $0,253 \mathrm{Kg}$ | $1,03 \mathrm{Kg}$ | $0,78 \mathrm{Kg}$ | $7,0 \mathrm{Kg}$ |




Fig. 2. The workplace of the payload center of gravity

## Studying the center of gravity chang

Through the study of the workspace changes of the pendulum motion in the 3D space, the payload's center of gravity will draw in space a half sphere-like shape as in figure (2). Due to these changes the aircraft's dynamics model center of gravity will change accordingly through the axis of the aircraft body $X B, Y B, Z B$, as in figure (3).These changes as a whole are considered similar to half-sphere shape too. As it is noticed from the curves and as shown in the table (2), the maximum value reached by the elements of the centers of gravity of the model. The process of studying the workspace of the movement of the centers of gravity was made using computerized simulation.

(b). disturbances along YB

(d). disturbances in 3-D

Fig. 3. The disturbances in the center of gravity

Table 2. Maximum values reached by the elements of the disturbances in the CoG

|  | $X_{B}$ | $Y_{B}$ | $Z_{B}$ |
| :--- | :---: | :---: | :---: |
| $\operatorname{Max}(\mathrm{~m})$ | 0,2602 | 0,005475 | 0,3093 |
| $\operatorname{Min}(\mathrm{~m})$ | $-0,3173$ | $-0,2359$ | $-0,2683$ |

## Studying the inertia moment

The inertia mass moment of complex shaped bodies is calculated using experimental methods, which in turn
enter into many calculations within the equations of the aircraft dynamics.

Here we will adopt mathematical methods in addition to simulation in the process of estimating the moment, where the inertia moment of the overall aircraft model is given according to the formula described in the relation bellow, through which the mathematical study has been simplified by dividing the overall aircraft model into several parts and each part was studied separately.

Where our goal is to determine the amount of inertia moment of the overall aircraft model according to the basic coordinate system $X B, Y B, Z B[6]$.

$$
\begin{equation*}
J=J_{\text {Body }}+J_{\text {Link } 1}+J_{\text {Link } 2}+J_{\text {Joint }}+J_{\text {Load }}, \tag{2}
\end{equation*}
$$

where $J$ : Inertia moment matrix of the overall aircraft dynamics model. $J_{\text {Body }}$ : Inertia moment matrix of the aircraft body. $J_{\text {Joint }}$ : Inertia moment matrix of the joint. $J_{\text {Link } 1}$ : Inertia moment matrix of the first link. $J_{\text {Link } 2}$ : Inertia moment matrix of the second link. $J_{\text {Load }}$ : Inertia moment matrix of the load. Where the inertia moment matrix takes, the general form shown in the equation:

$$
\begin{equation*}
J=\left[J_{x x}-J_{x y}-J_{x z} ;-J_{y x} J_{y y}-J_{y z} ;-J_{z x}-J_{z y} J_{z z}\right], \tag{3}
\end{equation*}
$$

The inertia for each part of the aircraft and the pendulum is studied according to several theories. Including the theory of parallel axis of Huygens-Steiner [8], which is concerned with studying the new inertia moment of the studied part relative to a new axis parallel to its axis and shifted from it by a specific distance, as in the equation bellow. Through our study, we will decide the inertia moment of each part of the overall model according to the coordinates system $X B, Y B, Z B$.

$$
\begin{equation*}
J_{0}^{*}=J_{0}+M \cdot d^{2} \tag{4}
\end{equation*}
$$

where $J_{0}^{*}$ is the new inertia mass moment around a new axis of rotation parallel to a local axis of rotation passing through the center of gravity and shifted from it by a distance $d$, while $M$ is the mass of the studied body, and $d$ is the distance between the local axis of rotation of the body passing through its center of gravity and the new axis under study, this parameter is important as it is constantly changing as a result of the difference in angles of the pendulum movement, which in turn lead to a change in the coordinates of the new overall aircraft mass center and thus the emergence of ongoing changes in the inertia moment of the studied body with time according to the coordinates system axes of the overall aircraft dynamics model. When the coordinates system and axes are nonparallel between the studied part and the overall model, the inertia moment is then determined according to a rotation axis having different directions with angles ( $\alpha$, $\beta, \gamma)$ so that the direction of its axes at each value of the pendulum angles is parallel to the direction of the dynamics model coordinates system axes and passes through the mass center of the studied part by the mathematical relationship, [7, 9, 10] See Figure (4).
$J_{0}=J_{x x} \alpha^{2}+J_{y y} \beta^{2}+J_{z z} \gamma^{2}-2 J_{x y} \alpha \beta-2 J_{x z} \alpha \gamma-2 J_{y z} \beta \gamma$,
where $J_{0}$ : new inertia moment of the studied part according to new axes parallel to the output coordinates system by the angles $\alpha, \beta, \gamma$, as shown in Figure (4). While $J_{x x}$, $J_{y y}, J_{z z}, J_{x y}, J_{x z}, J_{y z}$ are the elements of the local inertia moment matrix of the studied part.

The Curves in figures (5) to (10) illustrate the disturbances in the inertia moments of the aircraft overall dynamic model relative to the range of the pendulums angles change. As it is noticed from the curves, table (3)
shows the maximum reached value by the elements of the inertia moments matrix of the model. Table (4) shows the errors between the calculated results based on Solid Works program and the theoretical calculations of the values of specific samples of the pendulum angles in order to illustrate the difference between the derived values by simulation and the theoretical study.


Fig. 4. Angles of difference
between the local axis of the studied body and the new axis
Table 3. Maximum values reached by the elements of the inertia moments matrix of the overall model

|  | $J_{x x}$ | $J_{y y}$ | $J_{z z}$ | $J_{x y}$ | $J_{x z}$ | $J_{y z}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Max | 25,21 | 43,06 | 47,31 | 5,872 | 7,179 | 7,44 |
| Min | 10,67 | 28,42 | 34,65 | $-6,24$ | $-7,45$ | $-7,02$ |

Table 4. Errors between the calculated results based on SolidWorks program and the theoretical calculations

| Moments <br> of inertia <br> $\mathrm{kg} \cdot \mathrm{mm}^{2}$ | No.1 | No.2 | No.3 |
| :---: | :---: | :---: | :---: |
|  | $\theta_{1}^{\circ}=0$ | $\theta_{1}^{\circ}=-5$ | $\theta_{1}^{\circ}=216$ |
|  | $\theta_{3}^{\circ}=-31$ | $\theta_{2}^{\circ}=-158$ | $\theta_{2}^{\circ}=-107$ |
|  | Error $\%$ | $\theta_{3}^{\circ}=46.5$ | $\theta_{3}^{\circ}=-62$ |
| $J_{x x}$ | 0,0000 | 0,2125 | 0,2131 |
| $J_{y y}$ | 0,0026 | 0,1854 | 0,2225 |
| $J_{z z}$ | 0,0017 | 0,2692 | 0,0955 |
| $J_{y z}$ | 0,2319 | 1,6490 | 0,5049 |
| $J_{x y}$ | 1,9312 | 2,0046 | 3,1939 |
| $J_{x z}$ | 1,4129 | 0,3839 | 3,6553 |

## Conclusions

The results in this paper can determine a clear vision of the disorders affecting the dynamics of the aircraft motion in space, and help in implementing a model that is resistant to disturbances as much as possible in addition to designing an adaptive controller resistive to noise. Finally, we can analyze the general form of disturbances, which simulates the robotic arm motions and insert it into the aircraft's model equations of motion. These disturbances were presented as shown in figure (11) and were emulated based on the characteristics of the results of studies related to the change of inertia moments and center of gravity.


Fig. 11. Force and momentum disturbances resulted from the robotic arm

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