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ANALYSIS ROBUST STABILIZATION FOR MARKOV JUMP LINEAR SYSTEMS

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We are concerned with the problems of analysis stabilization and analysis robust stability for Continuous Time Markovian jump linear systems. The Markovian jump linear system includes parameter uncertainties both in the mode transition rate matrix and in the system matrices. Sufficient conditions are ensured to systems considered to be stable in the mean square stable are presented in the form of linear matrix inequalities. The conclusion of previous condition of robust stability for Continuous Time Markovian jump linear systems is presented in the form of a theorem. Sufficient condition for the design of controller's robust state-feedback where the closed-loop system quadratic mean square stable. The robust stabilization problem for Markovian jump linear systems was analyzed and state-feedback controller is designed such that the resulting closed-loop system is mean square stable. Finally, numerical example is provided to illustrate the effectiveness of the proposed theoretical results, the robust stabilizing controller for a Continuous Time Markovian jump linear system obtained by the MATLAB LMI Toolbox.

Keywords: Markov Jump Linear Systems, Robust stability analysis, linear matrix inequalities (LMIs).

Introduction

Markovian jump system (MJS) is a special class of dynamic systems subject which abrupt changes in their dynamics, and the model of system is model linear systems or nonlinear systems, Markov chain determined switching between the models [1]. The literature surrounding this topic is now extensive [2–6]. MJSs is special case of hybrid systems with the switching matrix governed by a Markovian chain. From a mathematical view, MJSLs can be regarded as a special class of stochastic linear systems with system matrices changing randomly at discrete- time points governed by a Markov process and remaining time- invariant between random jumps. Over the past decades, a great amount of attention has been paid to MJSs, due their wide applications in systems.

Applications of MJLSs in many real-world applications, economics, robotics, air vehicles, satellite dynamics, and wireless communication, among others such as economic system [7, 8], flight system [9, 10], robotic systems [11], power system [12, 13], communication system [14] and systems of networked control systems [15].

In recent years, to ease the practical application of MJLSs, considerable efforts have been made, and a lot of progresses have been made on topics such as: 1) modeling of MJSs; 2) control and filtering; 3) analysis stabilization and analysis stability robust; 4) Error detection and fault tolerance; 5) H_∞ Control; and so on.

Notations: We will denote by R^n the n-dimensional Euclidean space and by $\mathrm{B}(\mathrm{R}^n, \mathrm{R}^m)$ the norm bounded linear space of all $n \times m$ matrices with $\mathrm{B}(\mathrm{R}^n) = \mathrm{B}(\mathrm{R}^n, \mathrm{R}^n)$. For a matrix $A = \mathrm{B}(\mathrm{R}^n)$, A^T is the transpose of A. S_n is the set of all $n \times n$ symmetric matrices. $A \geq 0$ (resp., $A \leq 0$): will mean that the symmetric (resp., semi-negative definite) matrix $A = \mathrm{B}(\mathrm{R}^n)$ is positive semidefinite, and $A > 0$ (resp., $A < 0$): that is positive definite (resp., negative definite) matrix. $\mathbf{E}(\cdot)$ is the expectation operator. I is the $n \times n$ identity matrix.

Problem statement

Let the following Markovian jump linear systems (MJLS), defined on a complete probability space (Ω, F, P), are described as:

$$\dot{x}(t) = A(\theta(t))x(t) + B(\theta(t))u(t),\ t \geq 0. \quad (1)$$

Where $x(t) \in \mathrm{R}^n$ is standing for the state variable of the system, $u(t) \in \mathrm{R}^n$ is the control variable. We define the set $S = \{1, 2, \ldots, N\}$, $\{\theta(t),\ t \geq 0\}$ is a continuous-time Markov chain on the probability space, takes values on set S with transition probability matrix $\Pi = (\pi_{ij})_{N \times N}$ given by [16]

$$\mathrm{P}(\theta(t+h) = j \mid \theta(t) = i) = \begin{cases} \pi_{ij}h + o(h),\ i \neq j, \\ 1 + \pi_{ii}h + o(h),\ i = j. \end{cases} \quad (2)$$

And the notation $o(h)$ denotes a function on h, i.e., $h > 0$, $\lim\limits_{h \to 0} \dfrac{o(h)}{h} = 0$ and $\pi_{ij} \geq 0$ $(i, j \in S,\ i \neq j)$, represents the transition rate from i to j, which satisfies $\pi_{ii} = -\sum\limits_{j \neq i} \pi_{ij}$ for all $i \in S$. Let for $\Pi = (\pi_{ij})_{N \times N}$ the error between them is referred as to $\Delta\pi_{ij}$ which can take any value in $[-\varepsilon_{ij}, \varepsilon_{ij}]$.

Suppose $Q(t)$ denote to the second-order moments of the state vector $x(t)$, i.e. $Q(t) = \mathbf{E}(x(t) \cdot x^T(t))$ and define $Q_i(t) = \mathbf{E}(x(t) \cdot x^T(t) \mid \theta(t) = i)$. Then, given the theory of total probability, $Q(t) = \sum\limits_{i=1}^{N} Q_i(t)$. The N

differential equations governed of the components of the second-order moment

$$\dot{Q}_j(t) = A_j Q_j(t) + Q_j(t) A_j^T + \sum_{i=1}^{N} \pi_{ij} Q_i(t). \quad (3)$$

The following properties are easy to see hold.

Lemma 1. Let the solution $Q_j^a(t)$ of (3) with initial condition $Q_j^a(0)$. With initial condition $Q_j(0) = \alpha Q_j^a(0)$ the solution of (3) is $Q_j(t) = \alpha Q_j^a(t)$, $\forall t > 0$.

Lemma 2. If, at a certain time instant τ, $\dot{Q}_j(t) < 0$, $\forall j \in S$ then $Q_j(t) < 0$, $\forall t > \tau,\ j \in S$.

Lemma 3. Let $Q_j^a(t)$ and $Q_j^b(t)$ be the solution of (3) with initial condition $Q_j^a(0)$ and $Q_j^b(0)$ respectively. If $Q_j^a(0) < Q_j^b(0)$, $\forall j \in S$, then $Q_j^a(t) < Q_j^b(t)$, $\forall j \in S$.

Definition 4. The MJLS (1) is said to be mean square stable (MS-stable) if $\lim_{t\to\infty} \mathbf{E}\left(x(t)^2\right) = 0$ for any initial condition $x(0)$ and any initial probability distribution π_0. Moreover, the MJLS (1) is exponentially mean square stable (EMS-stable) if there exist positive real scalars α and β such that $\mathbf{E}\left(x(t)^2\right) < \alpha \cdot e^{-\beta t}$.

Proposition 5. The Markovian jump linear system of (1) is MS-stable if and only if the coupled LMIs

$$A_j Q_j(t) + Q_j(t) A_j^T + \sum_{i=1}^{N} \pi_{ij} Q_i(t)$$

are possible for a matrices $\left\{ Q_i : Q_i \in S^{n\times n} \right\}$.

Theorem 6. MJLS (1) is MS-stable if there exist positive definite matrices $Q_i,\ i \in S$ such that the following LMI's are satisfied

$$A_j Q_j(t) + Q_j(t) A_j^T + \sum_{i=1}^{N} \pi_{ij} Q_i(t) < 0. \quad (4)$$

Robust stability analysis

The aim of this part is to analysis robust stabilization problem for MJLS (1), we develop a new condition for analyze the robust stability property by used LMIs, and design a state-feedback controller such that closed-loop system is quadratically MS-stable.

Theorem 7. Markovian jump system (1) (for initial condition $u(t) \equiv 0$) is quadratically mean square stable if there are: $\left\{ P_i : P_i \in S^{n\times n},\ i \in S \right\}$, $\left\{ \lambda_i : \lambda_i \in \mathrm{R}^+,\ i \in S \right\}$, $\left\{ \lambda_{ij} : \lambda_{ij} \in \mathrm{R}^+,\ i, j \in S,\ i \neq j \right\}$ and E_i, H_{ai}, H_{bi} are known constant real matrices of appropriate dimensions, where

$$\begin{pmatrix} Q_i & P_i E_i & M_i \\ E_i^T P_i & -\lambda_i I & 0 \\ M_i^T & 0 & -\Lambda_i \end{pmatrix} < 0 \text{ for all } i \in S. \quad (5)$$

Such that

$$Q_i = A_i P_i + P_i A_i^T + \sum_{j=1}^{N} \pi_{ij} P_j + \frac{1}{4} \sum_{\substack{j=1,\\ j\neq i}}^{N} \lambda_{ij} \varepsilon_{ij}^2 I + \lambda_i H_{ai}^T H_{ai},$$

$$M_i = \left(P_i - P_1, \ldots, P_i - P_{i-1}, P_i - P_{i+1}, \ldots, P_i - P_N\right),$$

$$\Lambda_i = \operatorname{diag}\left(\lambda_{i1} I, \ldots, \lambda_{i(i-1)} I, \lambda_{i(i+1)} I, \ldots, \lambda_i I\right).$$

Inequality (5) is Schur complement equivalence

$$A_i P_i + P_i A_i^T + \sum_{j=1}^{N} \pi_{ij} P_j + \lambda_i H_{ai}^T H_{ai} + \frac{1}{\lambda_i} P_i E_i E_i^T + \sum_{\substack{j=1,\\ j\neq i}}^{N} \left(\frac{\lambda_{ij}}{4} \varepsilon_{ij}^2 I + \frac{1}{\lambda_{ij}} \left(P_j - P_i\right)^2 \right) < 0. \quad (6)$$

Let the state-feedback control law

$$u(t) = K(\theta(t)) \cdot x(t). \quad (7)$$

Where $K_i = K(\theta(t) = i) \in \mathrm{R}^{m\times n}$ is the controller which determined. The closed-loop system is

$$\dot{x}(t) = \begin{Bmatrix} A(\theta(t)) + B(\theta(t)) \cdot K(t) + E(\theta(t)) \cdot F(\theta(t)) \times \\ \times \left[\mathrm{H}_a(\theta(t)) + \mathrm{H}_b(\theta(t)) \cdot K\left(H_a(\theta(t))\right) \right] \end{Bmatrix} \times x(t). \quad (8)$$

The following theorem solves the robust stabilization problem (RSP) for MJLS (1).

The following theorem offers a condition of robust stability for MJLS (1).

Theorem 8. Let MJLS (1), and suppose a state-feedback control law (7) where the closed-loop system (8) is quadratically MS-stable if there exist sets of matrices satisfying the coupled of LMIs

$$\begin{pmatrix} Q_{1i} & \left(H_{ai} X_i + H_{bi} Y_i\right)^T & X_i \\ H_{ai} X_i + H_{bi} Y_i & -\alpha_i I & 0 \\ X_i^T & 0 & -Z_i \end{pmatrix} < 0, \quad (9)$$

$$\begin{pmatrix} Q_{2i} & M_i \\ M_i^T & -\Lambda_i \end{pmatrix} \le 0, \quad (10)$$

with equality constraints:

$$P_i X_i = I,\ V_i Z_i = I, \quad (11)$$

$$Q_{1i} = \left(A_i X_i + B_i Y_i\right)^T + \left(A_i X_i + B_i Y_i\right) + \alpha_i E_i E_i^T,$$

$$Q_{2i} = -V_i + \sum_{j=1}^{N} \pi_{ij} P_j + \frac{1}{4} \sum_{\substack{j=1,\\ j\neq i}}^{N} \lambda_{ij} \varepsilon_{ij}^2 I.$$

The controller (7) is given by the form $K_i = Y_i P_i$.

Proof. From the inequality (6), and put $V_i = Z_i^{-1}$ for all i we find

$$\sum_{j=1}^{N} \pi_{ij} P_j + \sum_{\substack{j=1,\\ j\neq i}}^{N} \left(\frac{\lambda_{ij}}{4} \varepsilon_{ij}^2 I + \frac{1}{\lambda_{ij}} \left(P_j - P_i \right)^2 \right) \le V_i .$$

The previous inequality is equivalent to inequality (10) in form Schur complement equivalence. Replacing A_i and H_{ai} in (6) with $A_i + B_i K_i$ and $H_{ai} + H_{ai} K_i$ respectively, we find

$$P_i \left(A_i + B_i K_i \right) + \left(A_i + B_i K_i \right)^T P_i + \frac{1}{\lambda_i} P_i E_i E_i^T P_i +$$
$$+ \lambda_i \left(H_{ai} + H_{bi} K_i \right)^T \left(H_{ai} + H_{bi} K_i \right) + V_i < 0.$$

Therefore, the closed-loop system (8) is quadratically MS-stable if all the above inequality are holds.

We apply the changes of variables $X_i = P_i^{-1}$, $Y_i = K_i X_i$, $\alpha_i = \lambda_i^{-1}$ after multiply by P_i^{-1} to both sides of the above inequality yield

$$\left(A_i X_i + B_i Y_i \right) + \left(A_i X_i + B_i Y_i \right)^T + \alpha_i E_i E_i^T +$$
$$+ \alpha_i^{-1} \left(H_{ai} + H_{bi} K_i \right)^T \left(H_{ai} + H_{bi} K_i \right) + X_i V_i X_i < 0,$$

which is equivalent to (9) by form Schur complement equivalence again.

Numerical example

We give an example of simulation to elucidate the usefulness and of the theory developed in this paper. Focuses on the design of a robust stabilizing controller to the MJLS.

Consider a Markovian jump linear system (1). The system data with the initial conditions of (1) are as follows:

$$A_1 = \begin{bmatrix} 0.176 & 0.784 \\ 0.926 & 0.136 \end{bmatrix}, \; A_2 = \begin{bmatrix} 0.547 & 0.127 \\ 0.616 & 0.965 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.299 \\ 0.447 \end{bmatrix}, \; B_2 = \begin{bmatrix} 0.741 \\ 0.795 \end{bmatrix},$$

$$\Pi = \begin{bmatrix} -6.700 & 6.700 \\ 6.918 & 0.136 \end{bmatrix}, \; x_0 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

The robust stabilizing controller for a Markovian jump system can be obtained by the MATLAB LMI Toolbox:

$$k_1 = \left[-1.607 \; -1.489 \right], \; k_2 = \left[-0.310 - 2.679 \right].$$

The resulting closed-loop system when applying this controller makes become mean square stable (Fig. 1).

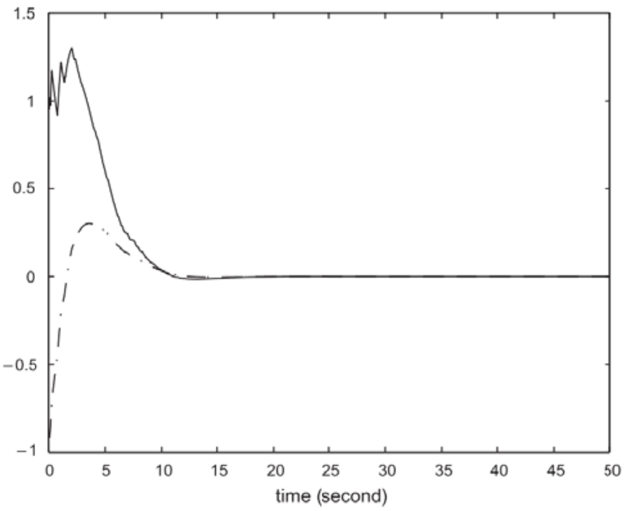

Fig. 1. MS- stable closed-loop system

Conclusion

In this paper reviewed the problems of stability and stabilization of Markovian jump linear systems. The LMI-based sufficient conditions ensuring systems considered to be mean square stable. Numerical example is provided to show the applicability of the developed method for stabilizing controller of MJLS.

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Анализ робастной стабилизации линейных систем с марковскими скачками

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В статье рассматривается проблема анализа устойчивости и робастной стабилизации непрерывных линейных систем с марковскими скачками. Линейная система с марковскими скачками содержит неопределенные параметры как в переходных матрицах состояний, так и в матрицах системы. Достаточные условия, гарантирующие асимптотическую устойчивость рассматриваемой системы в среднеквадратическом смысле, представлены в виде линейных матричных неравенств. Вывод предложенного выше условия робастной устойчивости для непрерывных линейных систем с марковскими скачками представлен в виде теоремы. Достаточное условие позволяет проектировать контроллер с робастной обратной связью таким образом, что полученная замкнутая система является устойчивой в среднеквадратическом смысле. Проведен анализ проблемы робастной стабилизации для линейных систем с марковскими скачками и спроектирован контроллер с обратной связью таким образом, что полученная замкнутая система является устойчивой в среднеквадратичеcком смысле. В конце статьи, для иллюстрации эффективности предлагаемых теоретических результатов, представлен численный пример робастного стабилизирующего контроллера для линейных систем с марковскими скачками, полученный с помощью MATLAB LMI Toolbox.

Ключевые слова: линейные системы с марковскими скачками, анализ робастной стабилизации, линейные матричные неравенства.

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