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## MODELING AND PREDICTIVE CONTROL OF NONLINEAR COUPLED AND UNDERACTUATED DYNAMICS OF A HEXACOPTER

### Introduction

Nowadays, reducing the human factor in flights is a common topic in aerospace applications, for making the loss of human lives and economic costs as low as possible. As a consequence of this pursuit, there is an overwhelming interest in the Unmanned Aerial Vehicles (UAVs). When there are sufficient measurements the necessary states of an autonomous aerial vehicle can be estimated via the Kalman Filter (KF) [1]. Unmanned Aerial Vehicles (UAVs) fly at very low speeds and Reynolds numbers, have nonlinear coupling and tend to exhibit time-varying characteristics [1]. In order to control such vehicles it becomes necessary to design and develop robust and adaptive controllers and hence, identify the system. State and parameter estimation, which are an integral part of system identification, has been carried out on various flight data and simulations as evident from literature [2]. Some researchers, presented attitude estimation algorithms based on Kalman filter such as SAKF [3] and RAKF [4], for UAV problems when noise statistical characteristics are unknown, and time-varying vibrations are the main disturbance source, also for problems against sensor/actuator fault of the system. Others papers presented the sliding mode and high-order sliding mode respectively like an observer [5, 6] in order to estimate the unmeasured states and the effects of the external disturbances such as wind and noise. In [7], the unknown parameter of the quadrotor are identified using state estimation method with the implementation of Unscented Kalman Filter (UKF). Most researchers used

estimation techniques to identify the parameters in unknown systems, while some researchers [8, 12, 14] mentioned estimation in order to predict the signals. The main objective is to estimate the altitude of the UAV, in order to use this estimated value as predictive feedback signals for UAV altitude stabilization. This value cannot be estimated using a single sensor because the fact that each sensor has its own problem [8]. The performance of the proposed KF is investigated using simulation for state estimation procedure of an Unmanned Aerial Vehicle [4, 8]. The paper proceeds as follows. In Section 2 the flight dynamics model of the UAV is given. In Section 3, UAV state estimation and predictive control are proposed. In Section 4, the simulation is carried out followed by a discussion. Section 5, gives a brief summary of the obtained results and concludes the paper.

### The UAV dynamics

Newton-Euler equations were used. In order to make the modeling, some assumptions have been made, taking into consideration that the hexacopter is a rigid body and has a symmetrical structure. The motion can be decomposed into translational and rotational components. Therefore, the equations with respect to the body frame are as derived in [9] and [10]:

$$\begin{cases} \dot{u} = -\frac{k_t}{m}u + g \cdot \sin \theta - (qw - vr) + \frac{F_{d1}}{m} \\ \dot{v} = -\frac{k_t}{m}v - g \cdot \sin \varphi \cdot \cos \theta - (ru - pw) + \frac{F_{d2}}{m} \\ \dot{w} = \sum_{i=1}^6 |T_i| - \frac{k_t}{m}w - g \cdot \cos \varphi \cdot \cos \theta - (pv - qu) + \frac{F_{d3}}{m} \end{cases} \quad (1)$$

$$\left\{ \begin{aligned} \dot{p} &= -\frac{\sqrt{3}}{2J_x} l (|T_3| - |T_4| - |T_5| + |T_6|) - \\ &\quad - \frac{k_r}{J_x} p - qr \frac{(J_z - J_y)}{J_x} + \frac{M_{d1}}{J_x} \\ \dot{q} &= -\frac{1}{2J_y} l (|T_3| - |T_4| + |T_5| - |T_6| + 2|T_1| - 2|T_2|) - \\ &\quad - \frac{k_r}{J_y} q - pr \frac{(J_x - J_z)}{J_y} + \frac{M_{d2}}{J_y} \\ \dot{r} &= -\frac{\rho C_Q A R^3}{J_z} l (\omega_1^2 + \omega_4^2 + \omega_6^2 - \omega_2^2 - \omega_3^2 - \omega_5^2) - \\ &\quad - \frac{k_r}{J_z} r - pq \frac{(J_y - J_x)}{J_z} + \frac{M_{d3}}{J_z} \end{aligned} \right. \quad (2)$$

where  $m$  is the hexacopter's total mass,  $k_t$  is the constant of aerodynamic force,  $g$  is the gravity constant,  $F_{dist} = [F_{d1} \ F_{d2} \ F_{d3}]^T$  is the disturbance force along the axis,  $T_i$  and  $\omega_i$  (where  $i = 0 \dots 6$ ) are the thrust and angular moments of the motors,  $J_x, J_y$  and  $J_z$  are inertial moments of the rigid body along axes,  $k_r$  is the constant of aerodynamic moment,  $l$  is the distance from CG to the center motors,  $C_Q$  is the motor's torque coefficient,  $\rho$  is the air density,  $A$  is the disc area,  $R$  is the disc radius, and  $M_{dist} = [M_{d1} \ M_{d2} \ M_{d3}]^T$  is the disturbance moment

along the axis. The equations of motion that govern the translational and rotational motion for the hexacopter with respect to the inertial (Earth) frame are [9, 10]:

$$\ddot{\xi} = [\ddot{x} \ \ddot{y} \ \ddot{z}]_E^T = C_b^n \cdot [\dot{u} \ \dot{v} \ \dot{w}]_B^T \quad (3)$$

$$\ddot{\sigma} = [\ddot{\phi} \ \ddot{\theta} \ \ddot{\psi}]_E^T = S \cdot [\dot{p} \ \dot{q} \ \dot{r}]_B^T \quad (4)$$

The angular position of the body frame with respect to the inertial one is defined by Euler angles: roll  $\phi$ , pitch  $\theta$  and yaw  $\psi$ . These together form the vector:  $\sigma = [\phi \ \theta \ \psi]^T$  where  $\phi$  and  $\theta \in ]-\frac{\pi}{2}, \frac{\pi}{2}[$ ;  $\psi \in ]-\pi, \pi[$ . The inertial frame position of the vehicle is given by vector  $\xi = [x \ y \ z]^T$  [9, 7, 10]. While, the angular velocity is defined by the vector  $\Omega = [p \ q \ r]^T$ , and the linear velocity is defined by the vector  $V = [u \ v \ w]^T$  in the body frame. The transformation from the body frame to the inertial frame is realized by using the well-known rotation matrix  $C_b^n$  defined in [1, 9, 10], which is orthogonal and  $C_b^{nT} = C_b^{n-1} = C_n^b$ . In addition, the transformation matrix for angular velocities from the body frame to the inertial one is  $S$  as mentioned in [9, 10, 11]. The block diagram in Figure (1) shows the variables of the mathematical model of a hexacopter aircraft.

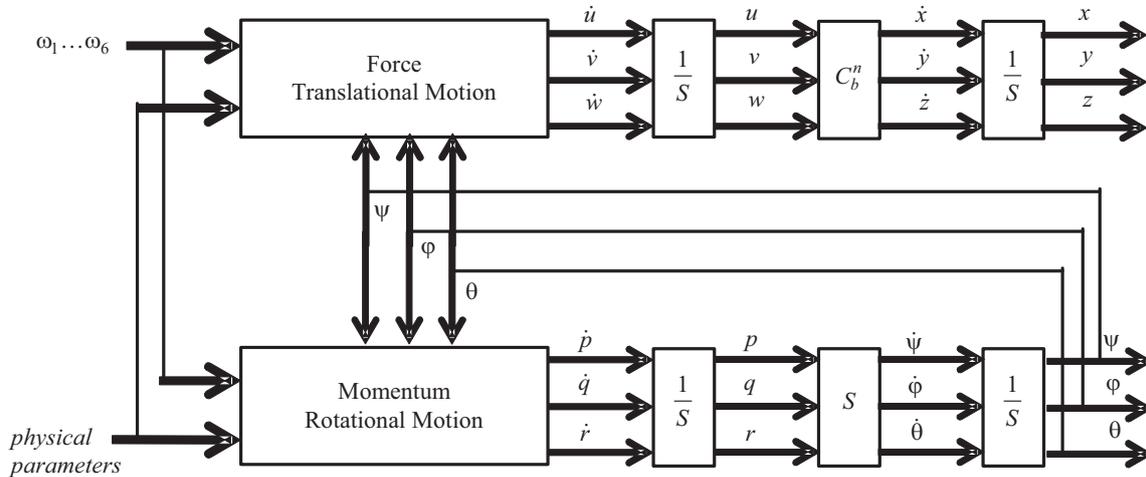


Figure 1. A mathematical model of Hexacopter

**UAV State Estimation and Predictive Control**

Large modeling errors may often appear when aircraft maneuvers. In order to decrease modeling errors [11], Subspace identification method via principle component analysis was used to identify a precise nonlinear model based on Kalman filter to estimate and predict the hexacopter's altitude from flight data [8, 14]. The estimated system is then used to cancel and minimize disturbances on the aircraft, as shown in figure (2) where the left side clarify the simple model without predictive

method while the right side shows how we used estimated model as feedback to enhance the response of the aircraft altitude by canceling the disturbances. The  $Y(t)$  is the altitude of the aircraft,  $Y'(t)$  is the disturbed altitude,  $X$  is the state space,  $u(t)$  is the control signal,  $e(t)$  is the error signal and  $r(t)$  is the altitude set-point. The effectiveness of the proposed method is evaluated comparing the simulation results with flight test data [12, 13].

From figure (2) it is clear that in the simple method, PID control loops were implemented to control the alti-

tude and velocity of a UAV rotorcraft. The PID gains can be selected precisely before the flight. No accurate model existed at first research stages. Tuning gains on a flying platform can be difficult because poor gains yield poor stability characteristics [13], which can lead to potentially destructive crashes. Once a set of acceptable gains has been found, it is not less difficult to ensure that they are optimal.

**Simulation and Discussion**

A LabVIEW simulation was conducted by using Runge-Kutta 2 method with a fixed step of 0.05 sec. As

shown in figure (3), it is clear that there is a long response time in addition to chattering and a big overshoot. These problems are results of the nonlinearity nature of hexacopter as well as coupling characteristic, which requires nonlinear methods to enhance the response. Predictive control technique was used in order to reduce the hexacopter problems, and the results were compared with those results taken from the simple method without inserting disturbances as shown in figure (2). Table shows the stability response of proposed controllers.

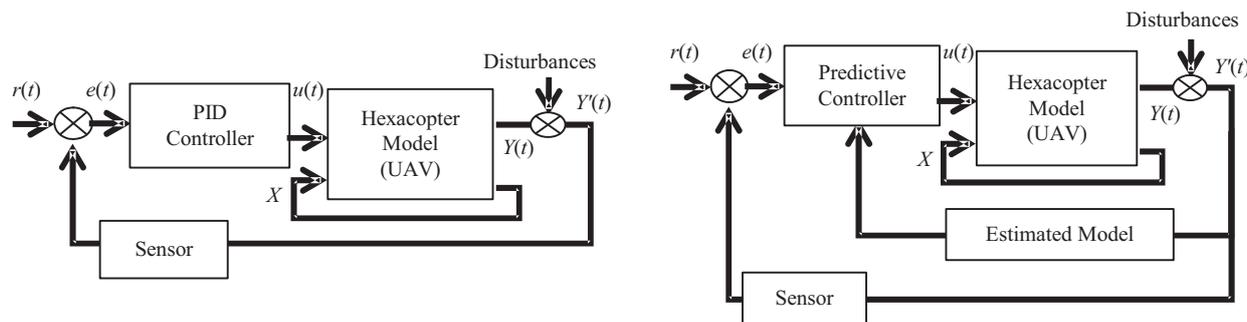


Figure 2. The proposed model of predictive control

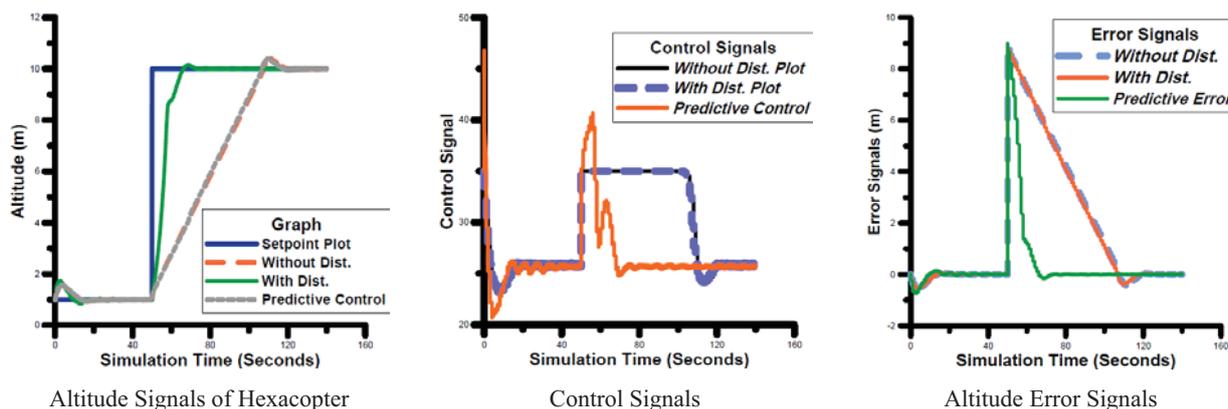


Figure 3

**Stability Response of Simple and Predictive Control**

Controller	Response Time (S)	Steady-State Error
PID controller	68	-0.044 (m)
Predictive Controller	23	0.0097 (m)

**Conclusion**

A real and complex dynamic model was considered, which addresses the nonlinearity, time variance, underactuation, and disturbance. The controller's parameters were tuned using Ziegler-Nichols algorithm to get the best performance and avoid the occurrence of vibrations in the output variables of the flying object as possible. In this paper, simple method and predictive method were used in altitude control, the predictive technique guarantees stability in presence of disturbances for nonlinear, coupled and underactuated systems compared with the simple method.

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