ОПУБЛИКОВАННЫЕ СТАТЬИ В ПЕРЕВОДЕ НА АНГЛИЙСКИЙ ЯЗЫК

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Replacement in a Plane Geared Linkage of High Kinematic Pairs with Lower Pairs*

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The analysis of constraints in plane mechanisms is an urgent problem in the theory of machines and mechanisms. Although kinematic pairs' classification has been known for a long time, the issue of the conjugation of links, being at the heart of the analysis and synthesis of mechanisms and machines, is of considerable theoretical and practical interest and continues to attract scientists.

One of the tasks that are solved in the process of analysis and synthesis of the structures of mechanisms is the replacement of higher kinematic pairs by lower ones. As a rule, such a replacement is made to identify kinematic chains of zero mobility, Assur's structural groups, in a mechanism. The replacement may also aim at obtaining the necessary kinematic relations. That is because specific computational difficulties hamper the kinematic analysis of chains with higher kinematic pairs due to the relative sliding and shape irregularity of mating surfaces. Yet, the use of replacements to obtain kinematic and transmission functions is difficult due to nonisomorphism of the equivalent mechanism. Simultaneously, for mixed-type mechanisms, which include geared linkages, the equivalent replacement will allow unifying the kinematic analysis methods.

The paper suggests the technology of replacing higher kinematic pairs with links with lower pairs as applied to a plane geared linkage. The technology is based on the properties of the involute of a circumference. The paper proved the structural and kinematic equivalence of such a replacement. The isomorphism of the equivalent linkage will enhance the kinematic analysis, make it possible using kinematic functions, and applying methods based on the instantaneous relative rotations of links, in particular, the Aronhold-Kennedy theorem. Another application of the replacement method presented in the paper will be the expansion of opportunities for identifying idle constraints in the mechanism.

Keywords: kinematic pair, constraint, equivalent mechanism, geared linkage.

Introduction

An important stage in the design of a mechanism is the development of a structural model which includes the physical characteristics of the links, the main ones being strength and rigidity, and the way in which the links are connected. At the two-link level, their interaction is modelled by kinematic pairs; at the higher level of the hierarchy, the mechanism is represented in the form of kinematic chains and Assur structural groups.

Kinematic pairs are classified by the number of constraints imposed on the relative motion of the links, as well as by the geometry of contact. In the higher kinematic pairs (according to the classification of F. Releaux), contact between the links occurs at a point or along a line, while in the lower pairs the conjugate profiles come into contact on the surface. As a rule, the kinematic analysis of chains with higher kinematic couples gives certain computational difficulties related to the possibility of relative sliding of links as well as irregularities in the shape of touching surfaces. The presence of dozens of geometric elements (lines and curves for flat conjugations, plane fragments and spherical, cylindrical, conical surfaces - in spatial conjugations) can lead to thousands of possible contact types in various combinations [1]. The determination of the geometrical characteristics of the motion of given points of a satellite in a planetary geared linkage also involves known mathematical difficulties [2, 3].

Lower couples are much simpler in this respect. When the contact goes to another place, the nature of the mutual arrangement and the shape of the touching surfaces in such pairs do not change, that is, the equations of constraints are considered to be unchanged. This is particularly important, for ex-

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ample, to ensure high accuracy in positioning the end effectors of manipulators [4].

Modern scientific theory of kinematic pairs was developed by L. T. Dvornikov and E. J. Zhivago [5]. Instead of the concept of an *element of a kinematic pair*, the authors introduce the concept of the *geometric element of a link*, a section of the surface of a link that is in direct contact with another link. This allows for a more precise differentiation of kinematic pairs based on the classification of simple surfaces whose Gaussian curvature sign does not change, as well as to consider different complexes of relative motions of the links.

Considering the kinematic pair as a set of constraints realized by a certain number of points of contact makes it possible to specify the principle of dividing the connections into active (necessary) and passive (redundant, idle). The problem of detecting and eliminating redundant constraints in plane linkages is topical and important for modern engineering. L. T. Dvornikov and L. N. Gudimova propose a method for eliminating redundant constraints in plane linkages, based on the solution of a system of two equations that describe the mechanical system under investigation simultaneously as coplanar and spatial simultaneously [6].

A new method of eliminating redundant constraints in single-circuit and multi-circuiut mechanisms is discussed in detail in the paper of V. N. Ermak [7]. According to this method, the mechanism is represented as the result of the imbrication of simple (without branching) open kinematic chains. In this case the structural formula is written in increments and applied to each chain separately.

The complexity of analyzing local mobility and redundant constraints in contours increases significantly for multi-circuit mechanisms such as planetary gear boxes [8]. In solving the problems of structural analysis of gear boxes it is possible to use the theory of graphs on the basis of which the interaction of contours is studied, the latter are formed by kinematic pairs whose classes must be previously justified [9].

Methods of structural synthesis of multi-circuit lincages are also being developed, in particular through the identification of the main topological characteristics of kinematic chains and the development of a generalized matrix model of the mechanism [10-12].

When studying the structure of plane mechanisms and solving problems of kinematics for the chosen positions, it is often convenient to replace higher kinematic pairs with links forming lower kinematic pairs. Since only rotational and prismatic lower kinematic pairs can be the part of the Assur structural groups, the equivalent replacement of higher kinematic pairs is often a prerequisite for structural analysis.

The method of equivalent mechanisms is popular when analyzing the structure of mechanisms, but due to some limitations, chief among which is the non isomorphic nature of the resulting structures, it is difficult to use this method for kinematic analysis [13]. For mixed-type mechanisms, which include geared lincages, it seems appropriate to reduce the structure under study to a single type of mechanism.

The aim of the research is to obtain the technology of replacement in planar geared linkages of higher kinematic pairs with lower ones and to prove the isomorphism of the resulting structure, that is, the independence of the lengths of the equivalent linkage from the relative positions of the links of the original mechanism.

Replacement technology

The problem of identifying plane toothed mechanisms of different types with linkages can have a number of applications, in particular for the kinematic and structural synthesis of gear mechanisms [14].

Consider the replacement of the higher kinematic pair formed by plane toothed contours. A plane geared linkage is shown in fig. 1, the instantaneous position of which is determined by the crank angle $\varphi_1 = 30°$ and the lengths of the links. The mechanism has the following geometry parameters: length of crank *1* is $l_{OA} = 0.05$ m; length of connecting rod *2* is $l_{AB} = 0.20$ m; length of rocker *3* is $l_{BC} = 0.24$ m; distance $l_{OC} = 0.20$ m. In the figure, the kinematic pairs formed by the movable links are indicated by double indexes.

The wheel z_2 is part of the connecting rod *AB*, while the wheel z_4 rotates around a fixed axis passing through the center of the hinge *C*. The number of teeth $z_2 = 25$; $z_4 = 35$, the radii of the pitch circles respectively are $r_2 = 0.1$ m and $r_4 = 0.14$ m. The teeth are cut without shift of the cutting tool. Crank *OA* rotates uniformly with an angular velocity of $\omega_1 = 7$ rad/s.

It is necessary to establish the technology of replacement of the higher kinematic pair at the point of contact of the toothed wheels 2 and 4 by the link forming two lower pairs. This replacement should make sense not only to analyze the structure, but also to find kinematic relationships. To check whether a replacement is correct, it is necessary to determine the mobility of the original mechanism and to calculate the output kinematic parameters, for example the angular velocity of the output link *4*.

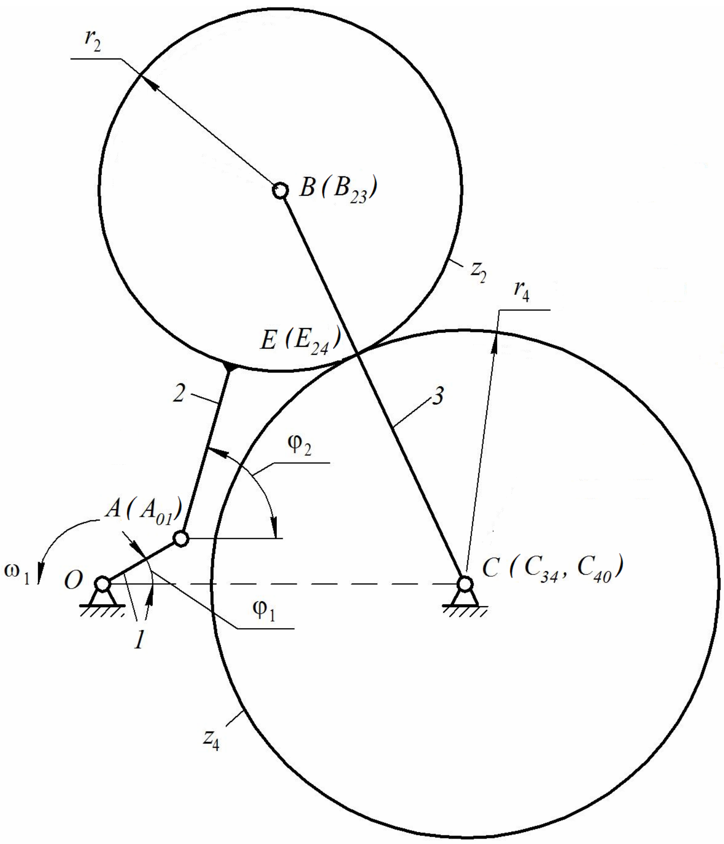

Fig. 1. Geared linkage

By the Chebyshev formula, the mobility of the plane mechanism is

$$W = 3n - 2p_5 - p_4.$$

In the mechanism under consideration, the number of movable links is $n = 4$, there are 5 rotational kinematic pairs of class 5 (O_{01}, A_{12}, B_{23}, C_{34}, D_{40}), i.e., $p_5 = 5$, one higher kinematic pair of class 4 (E_{24}), i.e., $p_4 = 1$. Hence, the mobility is

$$W = 3 \cdot 4 - 2 \cdot 5 - 1 = 1. \tag{1}$$

For the purpose of verifying kinematic equivalence, the instantaneous angular velocity of the toothed wheel *4* shall be determined. From the closed vector contour equation *OAB – OCB* (see fig. 2.) the projections on the coordinate axis gives

$$l_1 \cos\varphi_1 + l_2 \cos\varphi_2 = l_0 + l_3 \cos\varphi_3 \; ;$$
$$l_1 \sin\varphi_1 + l_2 \sin\varphi_2 = l_3 \sin\varphi_3 \; .$$

By differentiating the system of equations according to time and substituting given numerical values, we obtain equations for angular velocities. Solving these equaitions for a given position, we determine the instantaneous angular velocities of the links *2* and *3*: $\omega_2 = -2.67$ rad/s, $\omega_3 = -1.56$ rad/s (negative signs indicate clockwise rotation of the links).

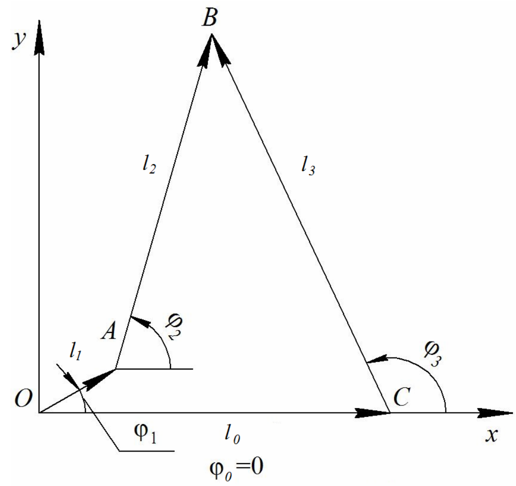

Fig. 2. Vector contour for initial mechanism

We use the values found to determine the velocity of the points E_2 and E_4 belonging to links *2* and *4,* taking the position *E* (see fig. 1) and forming the highest kinematic pair E_{24}.

For a point on the body moving with general plane motion the following vector equation applies

$$\vec{V}_{E_2} = \vec{V}_{B_2} + \vec{V}_{E_2B_2},$$

where

$\left|V_{E2B_2}\right| = r_2 \cdot \left|\omega_2\right| = 0.27$ m/s;

$\left|V_{B_2}\right| = l_{BC} \cdot \left|\omega_3\right| = 0.37$ m/s.

Taking into account the parallelism of the folding vectors and projecting the vector equation on an axis perpendicular to the *BC* line and directed upwards, we get (fig. 3)

$$V_{E_2} = V_{E_4} = 0.37 - 0.27 = 0.1 \text{ m/s}.$$

Then the angular speed of the gear wheel *4* is

$$\omega_4 = -0.71 \text{ rad/s}. \qquad (2)$$

In the last expression, the sign minus shows the direction of rotation, the inverse of the accepted directions of the angles, that is counterclockwise direction.

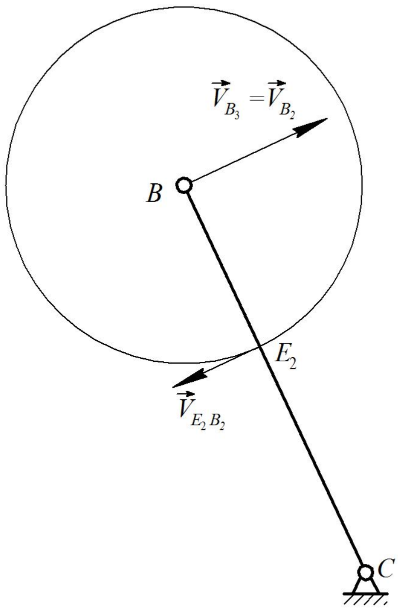

Fig. 3. Finding velocity of point E_2

In order to remove the highest kinematic pair E_{24}, a general normal to the conjugate profiles is drawn through this point and the corresponding radii of curvature is found. For involute toothed contours, this problem is solved relatively simply and precisely on the basis of the use of the properties of the evolute curve. For analytic determination of the coordinates of the points of an involute profile, it is necessary and sufficient to know only the radius of basic circle of a toothed wheel.

By the law of gearing, the contact normal to the profiles of the highest pair crosses the center-to-center line in the instant center of relative rotation of the links, which is the pitch point. The common normal to conjugate involute profiles of teeth, drawn through the pitch point, is tangent to the basic circles at the points N_2 and N_4, which are the centers of curvature of the involute profiles (fig. 4). These points serves as the centres of rotational lower kinematic pairs of the equivalent mechanism.

Thus, the proposed technology for replacing a higher kinematic pair in the engagement of two toothed contours is as follows. A common tangent to the basic circles should be drawn through the pitch point, and the lower rotational kinematic pairs should be arranged at the intersections of these lines (points N_2 and N_4 in fig. 4). The radii of the basic circles, carried out at the resulting points, make the pressure angle α_w with a line passing through the centers of the wheels. Features of the solution lead to some stable ratios of the lengths of the links and their mutual position: the distances between the joints placed in the wheel centres, and the joints of the equivalent link equal to the radii of the basic circles ($BN_2 = r_{b2} = r_2 \cos \alpha_w$, $CN_4 = r_{b4} = r_4 \cos \alpha_w$), link N_2N_4 is on the path of contact. The equivalent linkage is shown in fig. 5.

The validity of the replacement is checked according to the conditions mentioned above. The condition *for structural equivalence*: the mobility of the replacing mechanism is $W = 3n - 2p_5 = 3 \cdot 5 - 2 \cdot 7 = 1$. This result coincides with that obtained for the original mechanism (see (1)).

To check the fulfillment of the condition of *kinematic equivalence*, it is sufficient to find the angular velocity of the CN_4 link, which is a kinematic analogue of wheel *4* (fig. 6).

Vector equations for the velocities of point N_4, written through the parameters of links N_2N_4 and N_4C:

$$\vec{V}_{N_4} = \vec{V}_{N_2} + \vec{V}_{N_4N_2} = \vec{V}_{B_2} + \vec{V}_{N_2B_2} + \vec{V}_{N_4N_2}.$$

Given the specific location of the vectors, it is advisable to project the expression in the *n-n* direction (up):

$$V_{N_4} = V_{B_2} \cdot \cos 20° - V_{N_2B_2} =$$
$$= 0.37 \cdot \cos 20° - 0.1 \cdot \cos 20° \cdot 2.67 = 0.1 \text{ m/s}.$$

Angular velocity of link CN_4 equivalent to toothed wheel *4* is (with roundings)

$$\omega_4 = \frac{0.37 \cdot \cos 20° - 0.1 \cdot 2.7 \cdot \cos 20°}{0.14 \cdot \cos 20°} = 0.71 \text{ rad/s}.$$

This result coincides with that obtained for the original mechanism (see (2)), indicating the kinematic equivalence of the replacement.

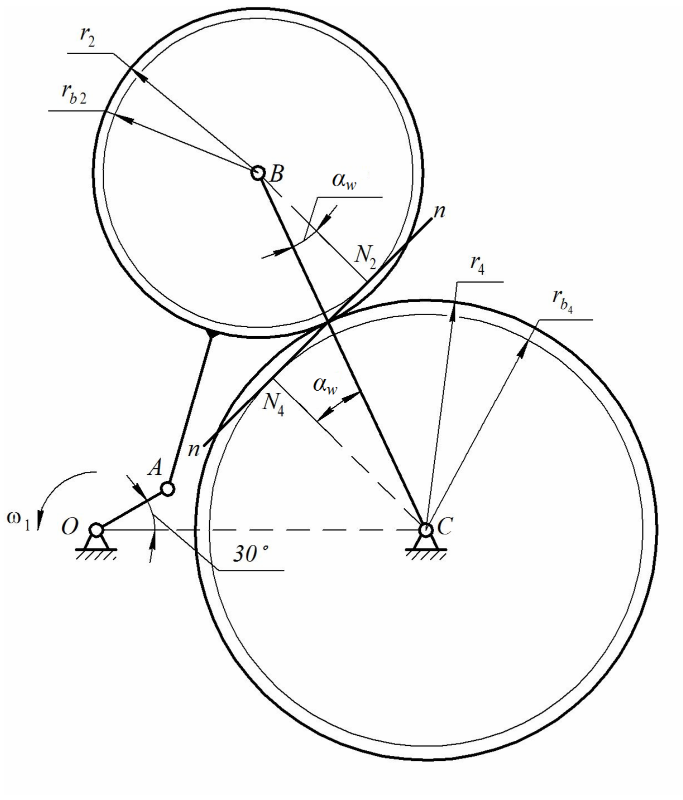

Fig. 4. Location of positions of the lower kinematic pairs (N_2, N_4), formed by the equivalent link

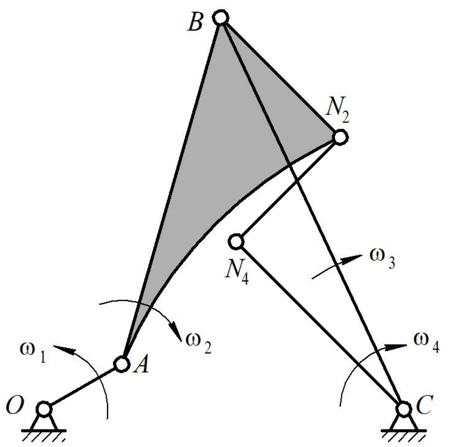

Fig. 5. Equivalent mechanism

Since the equivalent mechanism is a linkage containing only lower kinematic pairs of class 5, it can be divided into Assur's groups, with the crank OA_1 as the primary link (fig. 7).

The structural formula for equivalent second-class mechanism is:

$$I\left(\frac{1}{O_{01}^B}\right)\to II\left(\frac{2,3}{A_{12}^B B_{23}^B C_{30}^B}\right)\to II\left(\frac{4,5}{N_{25}^B N_{54}^B C_{40}^B}\right).$$

An important property of the obtained equivalent mechanism is its *isomorphism*, since the method of obtaining the lowest kinematic pairs ensures constant distances between points B, N_2, N_4, and C. The isomorphy of the equivalent linkage will allow for the application of analytical methods [15-17] for kinematic analysis of such mechanisms.

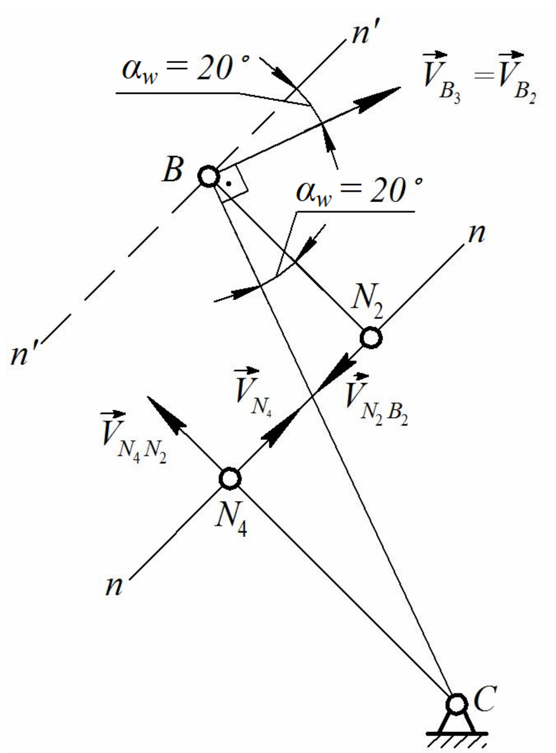

Fig. 6. Finding the velocity of the CN_4 link which is the kinematic analogue of wheel *4*

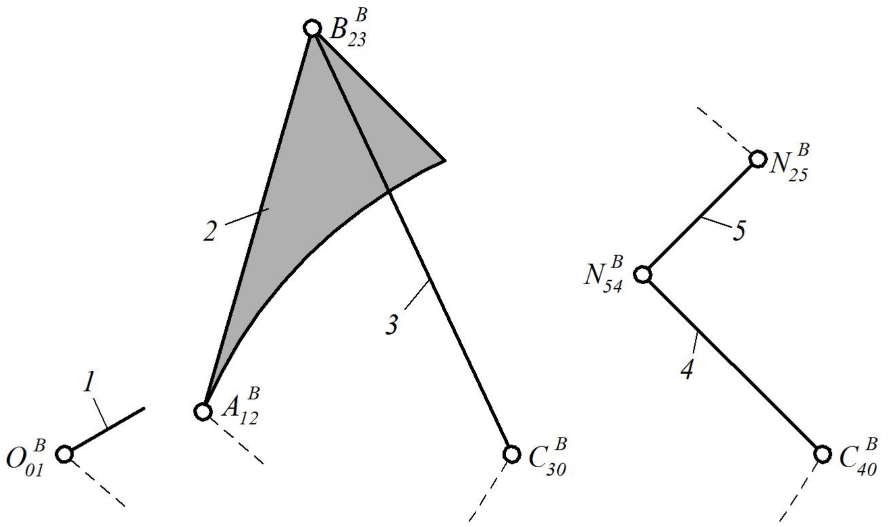

Fig. 7. Assur groups for the equivalent mechanism

Conclusion

The article proposes a technology for replacing higher kinematic pairs with lower pairs for plane geared linkages, based on the properties of the involute of a circle. The structural and kinematic equivalence of the replacement is proved on the example of a plane five-bar linkage. The proposed method will make it possible to obtain isomorphic equivalent linkages, which will considerably extend the possibilities of kinematic analysis by using kinematic functions and by considering instantaneous relative rotations of links with the application, in particular, Arongold-Kennedy theorem [18].

References

1. Müller A. [Higher-order constraints for higher kinematic pairs and their application to mobility and shakiness analysis of mechanisms]. *Meccanica*, 2017, vol. 52, pp. 1669-1684. DOI: 10.1007/s11012-016-0496-x.

2. Fedoseev G.N., Semin A.G., Korneenko D.V. [The study of the kinematic capabilities of the gear-lever mechanism with quasi-stops]. *Vestnik Vitebskogo gosudarstvennogo tekhnologicheskogo universiteta*, 2015, vol. 28, pp. 111-115 (in Russ.).

3. Suncov A.S. [Load distribution along the length of the teeth of the wheels of a two-satellite planetary gear with a prefabricated carrier]. *Vestnik IzhGTU imeni M.T. Kalashnikova*, 2016, vol. 19, no. 3, pp. 7-8 (in Russ.). DOI: 10.22213/2413-1172-2016-3-7-8.

4. Turygin Yu.V., Zubkova Yu.V., Speranskih T.N. [Development of a mathematical model of the movement of the robot output link based on the analysis of positioning errors]. *Vestnik IzhGTU imeni M.T. Kalashnikova*, 2018, vol. 21, no. 1, pp. 19-22 (in Russ.). DOI: 10.22213/2413-1172-2018-1-19-22.

5. Zhivago E.Ya. [History of the creation and classification of kinematic pairs]. *Teoriya mekhanizmov i mashin*, 2008, vol. 6, no. 2, pp. 26-34 (in Russ.).

6. Gudimova L.N., Dvornikov L.T. [Problems of elimination of excess bonds in planar articulated mechanisms]. *Fundamental'nye issledovaniya*, 2013, no. 6, pp. 24-32 (in Russ.).

7. Ermak V.N. [A new method for identifying and eliminating excess connections in multi-circuit mechanisms]. *Izvestiya vysshih uchebnyh zavedenij*, 2012, no. 1, pp. 18-23 (in Russ.).

8. Timofeev G.A., Samojlova M.V. [Structural analysis of a planetary mechanism with two floating links]. *Izvestiya vysshih uchebnyh zavedenij. Mashinostroenie*, 2017, no. 6, pp. 18-27 (in Russ.). DOI: 10.18698/0536-1044-2017-6-18-27.

9. Timofeev G.A., Podchasov E.O. [Using the graph method in the structural analysis of a wave gear]. *Vestnik nauchno-tekhnicheskogo razvitiya*, 2016, no. 6, pp. 23-31 (in Russ.).

10. Pozhbelko V.I., Kuc E.N. [System analysis and areas of application of multi-circuit linkage mechanisms with multiple joints in modern engineering]. *Izvestiya vysshih uchebnyh zavedenij*, 2020, no. 2, pp. 11-25 (in Russ.). DOI: 10.18698/0536-1044-2020-2-11-25.

11. Kuc E.N. [Structural synthesis of multi-circuit linkage mechanisms with multiple joints and the most complex double-joint link]. *Sovremennoe mashinostroenie. Nauka i obrazovanie*, 2019, no. 8, pp. 201-214 (in Russ.). DOI: 10.1872/MMF-2019-17.

12. Pozhbelko V. [Advanced technique of type synthesis and construction of veritable complete atlaces of multiloop F-DOF generalized kinematic chains]. *Mechanisms and Machine Science*, 2019, vol. 59, pp. 207-214. DOI: 10.1007/978-3-319-98020-1-24.

13. Gregorio Di R. [On Higher-Pair Modelling in Planar Mechanisms]. *Mechanisms and Machine Science*, 2019, vol. 73, pp. 863-870. DOI: 10.1007/978-3-030-20131-9-86.

14. Pozhbelko V.I. [Structural synthesis of lever and planetary mechanisms of a given level of complexity according to the universal table of standard structure codes]. *Izvestiya vysshih uchebnyh zavedenij. Mashinos-*

troenie, 2012, no. 4, pp. 13-29 (in Russ.). DOI: 10.18698/0536-1044-2012-4.

15. *Wang J.*, *Ting K.*, *Zhao D.* [Equivalent Linkages and Dead Center Positions of Planar Single-DOF Complex Linkages]. *Journal of Mechanisms and Robotics*, 2013, vol. 7, pp. 48–54. DOI: 10.1115/IMECE2013-62306.

16. *Kim M.*, *Han M.S.*, *Seo T.* [A new instantaneous center analysis methodology for planar closed chains via graphical representation]. *Control Autom. Syst.*, 2016, vol. 14, pp. 1528-1534. DOI: 10.1007/s12555-015-0066-3.

17. *Zhao J.*, *Feng Z.* [Kinematic Design of Geared Five-Bar Linkage]. *Design of Special Planar Linkages*, 2014, pp. 51-75.

18. *Balyakin V.*, *Krylov E.* [Cultural and Educational Significance of MMS Competitions for Future Engineers]. Springer Nature Switzerland AG 2019 J. C. García-Prada and C. Castejón (eds.): *New Trends in Educational Activity in the Field of Mechanism and Machine Theory*, MMS 64, 2019, pp. 38-48. DOI: 10.1007/978-3-030-00108-7-5.

Замена в плоском зубчато-рычажном механизме высших кинематических пар низшими парами

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Анализ связей в плоских механизмах является актуальной проблемой машиностроения. Несмотря на то, что классификация кинематических пар известна, проблематика сопряжения звеньев лежит в основе анализа и синтеза механизмов и машин, представляет значительный теоретический и прикладной интерес и продолжает привлекать ученых.

Одной из задач, которые решаются в процессе анализа и синтеза механизмов, является замена высших кинематических пар низшими. Как правило, такая замена производится в целях выделения кинематических цепей нулевой подвижности, структурных групп Ассура. Однако замена может также быть актуальной для кинематического анализа цепей с высшими кинематическими парами в силу определенных вычислительных трудностей, связанных относительными скольжениями звеньев, и нерегулярностью формы соприкасающихся поверхностей. При актуальности этой замены ее использование для получения кинематических и передаточных функций затруднено в силу ситуативности замены, неизоморфности эквивалентного механизма. В то же время для ряда механизмов смешанного типа, к которым относятся зубчато-рычажные механизмы, эквивалентная замена позволила бы унифицировать методы кинематического анализа.

Предложена технология замены высших кинематических пар звеньями с низшими парами, основанная на свойствах эвольвенты окружности. Доказана структурная и кинематическая эквивалентность такой замены для случая плоского зубчато-рычажного механизма. Показано, что изоморфность эквивалентного рычажного механизма позволит расширить возможности кинематического анализа, использовать кинематические функции, а также применять методы, основанные на рассмотрении мгновенных относительных вращений звеньев, в частности теорему Аронгольда – Кеннеди. Еще одним применением метода замены, который представлен в статье, будет расширение возможностей по выявлению контурных избыточных связей в механизме.

Ключевые слова: кинематическая пара, контурная связь, эквивалентный механизм, зубчато-рычажный механизм.

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